# A QUASI-STEADY METHOD OF DETERMINING THE THERMOPHYSICAL CHARACTERISTICS OF SOLIDS

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We consider a comparative quasi-steady method of determining thermophysical characteristics. Computational formulas are derived for the symmetric and nonsymmetric heating of plates. We present theoretical $a$- and $\lambda$-data for mycalex, derived on the basis of this method.

We know from [1] that methods based on a quasi-steady thermal regime enable us to derive the simplest and most exact calculational formulas for the determination of the thermophysical characteristics of solid materials.

Here we will examine one of the relative quasi-steady methods for the integrated determination of the coefficients of thermal diffusivity and thermal conductivity on the basis of the well-established $a$ and $\lambda$ of a standard material.

The physical model of the conductimeter consists of two unbounded plates, each of a different thickness, and each exhibiting diverse thermophysical characteristics. The plates are in thermal contact. Automatic programmed temperature regulators [2] are used to establish the linear temperature variations – with identical or differing heating rates – on the external side surfaces of these plates. It is assumed that $a$ and $\lambda$ are independent of temperature.

The temperature field $t_1(x, \tau)$ in the specimen under consideration and $t_2(x, \tau)$ in the standard are described for the one-dimensional problem by the equations

$$\begin{aligned} \frac{\partial t_1(x,\tau)}{\partial\tau} &= a_1\frac{\partial^2 t_1(x,\tau)}{\partial x^2}; \quad 0 \leqslant x \leqslant c; \\ \frac{\partial t_2(x,\tau)}{\partial\tau} &= a_2\frac{\partial^2 t_2(x,\tau)}{\partial x^2}; \quad -d \leqslant x \leqslant 0. \end{aligned} \tag{1}$$

Since the continuous solutions of these equations are analytical with respect to x, solution (1) can be presented in the form

$$t_1(x,\tau) = \sum_{m=0}^{\infty} A_m(\tau)\, x^m, \qquad 0 \leqslant x \leqslant c; \tag{2}$$

$$t_2(x,\tau) = \sum_{m=0}^{\infty} B_m(\tau)\, x^m, \quad -d \leqslant x \leqslant 0. \tag{3}$$

Having substituted (2) and (3) into (1), we derive the relationships for the coefficients

$$\begin{aligned} A_{m+2}(\tau) &= \frac{1}{a_1(m+2)(m+1)}\,\frac{dA_m(\tau)}{d\tau}; \\ B_{m+2}(\tau) &= \frac{1}{a_2(m+2)(m+1)}\,\frac{dB_m(\tau)}{d\tau}, \end{aligned} \tag{4}$$

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which can be brought to the form

$$A_{2m}(\tau) = \frac{1}{a_1^m (2m)!} \frac{d^m A_0(\tau)}{d\tau^m};$$
$$A_{2m+1}(\tau) = \frac{1}{a_1^m (2m+1)!} \frac{d^m A_1(\tau)}{d\tau^m} \tag{5}$$
$$(m = 1, 2, 3, \ldots).$$

Analogous expressions can also be derived for the coefficients

$$B_{2m}(\tau) \text{ and } B_{2m+1}(\tau).$$

We determine the coefficients $A_0(\tau)$, $A_1(\tau)$, $B_0(\tau)$, and $B_1(\tau)$ from the boundary conditions.

It follows from the condition of equality for the temperatures and heat flows in the plane of contact between the specimen and the standard, when x = 0, that

$$B_0(\tau) = A_0(\tau) \text{ and } \lambda_2 B_1(\tau) = \lambda_1 A_1(\tau), \tag{6}$$

$A_0(\tau)$ and $B_0(\tau)$ being determined from the boundary conditions

$$t_1(c, \tau) = T_1(\tau); \quad t_2(-d, \tau) = T_2(\tau). \tag{7}$$

The use of the exact solution for the determination of the thermophysical coefficients [1] led to the need of solving the complex transcendental equation which, in the final analysis, had to be solved with a rather arbitrary approximation. In addition, for practical purposes, i.e., the derivation of the computational formulas for the determination of $a$ and $\lambda$, the exact solution of the equation is not yet an exact solution for the stated problem, since the equation itself is a result of a somewhat schematized approach to the physical phenomenon.

A reasonable and approximate solution may therefore serve as an appropriate solution for the problem stated here.

Let us determine the approximate solution of the problem in the form of segments from series (2) and (3). The selection of the approximate solution affects the determination of the functions $A_0(\tau)$ and $A_1(\tau)$ from (7), and since the series coefficients are determined in terms of derivatives of these functions, the degree of approximation depends significantly on the choice of the approximate solution.

We will therefore set up an approximate a priori solution, and with this solution we will subsequently obtain an evaluation for the degree of the approximate solution and the limits of its applicability.

Let us assume that the temperature fields in the specimen and in the standard have been sufficiently well defined by the lowest powers in expansions (2) and (3), since the thickness of the specimen and standard are small in comparison with their heating surfaces. In the determination of the approximate solution we will therefore limit ourselves to the three terms in expansions (2) and (3):

$$t_1(x, \tau) = A_0(\tau) + A_1(\tau)x + \frac{x^2}{2a_1} A_0'(\tau), \tag{8}$$

$$t_2(x, \tau) = A_0(\tau) - \frac{\lambda_1}{\lambda_2} A_1(\tau)x + \frac{x^2}{2a_2} A_0'(\tau). \tag{9}$$

Substituting (7) into these equations and eliminating $A_1(\tau)$, for the function $A_0(\tau)$ we derive the differential equation:

$$(c\lambda_2 + d\lambda_1) A_0(\tau) + \frac{cd}{2}\left(\frac{c\lambda_1}{a_1} + \frac{d\lambda_2}{a_2}\right) A_0'(\tau) = c\lambda_2 T_2(\tau) + d\lambda_1 T_1(\tau). \tag{10}$$

Integrating Eq. (10) yields

$$A_0(\tau) = \left[A_0(0) + \frac{1}{\mu}\int_0^\tau [\lambda_2 c T_2(\xi) + \lambda_1 d T_1(\xi)] \exp k\xi \, d\xi\right] \exp(-k\tau), \tag{11}$$

where

$$\mu = \frac{cd}{2}\left(\frac{c\lambda_1}{a_1} + \frac{d\lambda_2}{a_2}\right); \quad k = \frac{\lambda}{\mu}; \quad \lambda = c\lambda_2 + d\lambda_1.$$

For our problem, in the case of a quasi-steady regime, the boundary conditions have the form

$$T_1(\tau) = T_0 + b_1\tau \text{ and } T_2(\tau) = T_0 + b_2\tau. \tag{12}$$

It is not difficult to demonstrate that $A_0(0) = T_0$, so that

$$A_0(\tau) = T(0,\tau) = T_0 + \frac{s}{\lambda}\tau - \frac{s}{\lambda k}\left[1 - \exp(-k\tau)\right]. \tag{13}$$

We determine $A_1(\tau)$ from (7):

$$A_1(\tau) = \frac{1}{c}\left(b_1 - \frac{s}{\lambda}\right)\tau + \frac{s}{\lambda}\left(\frac{1}{ck} - \frac{c}{2a_1}\right)\left[1 - \exp(-k\tau)\right] \tag{14}$$

Thus the approximate solution of Eqs. (1) is completely determined and has the form

$$\begin{aligned} t_1(x,\tau) = T_0 + \frac{s}{\lambda}\tau - \frac{s}{\lambda k} + \left[\frac{s}{\lambda}\left(\frac{1}{ck} - \frac{c}{2a_1}\right) + \frac{1}{c}\left(b_1 - \frac{s}{\lambda}\right)\right]x \\ + \frac{s}{2a_1}\lambda x^2 + \left[\frac{s}{\lambda k} - \frac{s}{\lambda}\left(\frac{1}{ck} - \frac{c}{2a_1}\right)x - \frac{sx^2}{2a_1\lambda}\right]\exp(-k\tau) \end{aligned} \tag{15}$$

and

$$\begin{aligned} t_2(x,\tau) = T_0 + \frac{s}{\lambda}\tau - \frac{s}{\lambda k} - \frac{\lambda_1}{\lambda_2}\left[\frac{s}{\lambda}\left(\frac{1}{ck} - \frac{c}{2a_1}\right)\right. \\ \left. + \frac{1}{c}\left(b_1 - \frac{s}{\lambda}\right)\right]x + \frac{s\lambda}{2a}x^2 + \left[\frac{s}{\lambda k} - \frac{s}{\lambda}\frac{\lambda_1}{\lambda_2}\left(\frac{1}{ck} - \frac{c}{2a_1}\right)x - \frac{s\lambda_1 x^2}{2a_2\lambda\lambda_1}\right]\exp(-k\tau). \end{aligned} \tag{16}$$

For the evaluation of the approximate solution of $t_1(x, \tau)$ we determine the exact solution of $t_1^*(x, \tau)$ of the thermal-conductivity equation (1) in the region $0 \le x \le c$, assuming that the functions $A_0(\tau)$ and $A_1(\tau)$ in (5) are precisely the same as in the approximate solution, i.e., they have the form of (13) and (14).

Then we have

$$\begin{aligned} A_0'(\tau) &= \frac{s}{\lambda}\left[1 - \exp(-k\tau)\right], \\ A_0^m(\tau) &= \frac{s}{\lambda k}(-k)^m \exp(-k\tau); \end{aligned} \tag{17}$$

$$\begin{aligned} A_1'(\tau) &= \frac{1}{c}\left(b - \frac{s}{\lambda}\right) + \frac{s}{\lambda}\left(\frac{1}{c} - \frac{ck}{2a_1}\right)\exp(-k\tau), \\ A_1^m(\tau) &= -\frac{s}{\lambda}\left(\frac{1}{ck} - \frac{c}{2a_1}\right)(-k)^m \exp(-k\tau); \end{aligned} \tag{18}$$

$$\begin{aligned} t_1^*(x,\tau) &= \sum_{m=0}^{\infty} A_m(\tau)x^m = \sum_{m=0}^{\infty} A_{2m}(\tau)x^{2m} + \sum_{m=0}^{\infty} A_{2m+1}(\tau)x^{2m+1} \\ &= T_0 - \frac{s}{\lambda k} + \frac{s}{\lambda}\tau + \left[\frac{s}{\lambda}\left(\frac{1}{ck} - \frac{c}{2a_1}\right) + \frac{1}{c}\left(b_1 - \frac{s}{\lambda}\right)\tau\right]x + \frac{sx^2}{2a_1\lambda} \\ &+ \frac{1}{6a_1c}\left(b_1 - \frac{s}{\lambda}\right)x^3 + \frac{s}{\lambda k}\exp(-k\tau)\sum_{m=2}^{\infty}\left(-\frac{k}{a_1}\right)^m \frac{x^{2m}}{(2m)!} \\ &- \left(\frac{1}{ck} - \frac{c}{2a_1}\right)\frac{s}{\lambda}\exp(-k\tau)\sum_{m=0}^{\infty}\left(-\frac{k}{a_1}\right)^m \frac{x^{2m+1}}{(2m+1)!}. \end{aligned} \tag{19}$$

Considering that

$$\sum_{m=2}^{\infty}\left(-\frac{k}{a_1}\right)^m \frac{x^{2m}}{(2m)!} = \sum_{m=2}^{\infty}\frac{(-1)^m\left(\sqrt{\frac{k}{a_1}}\,x\right)^{2m}}{(2m)!} = \cos\sqrt{\frac{k}{a_1}}\,x - 1 + \frac{1}{2}\left(\sqrt{\frac{k}{a_1}}\,x\right)^2$$

and

$$\sum_{m=2}^{\infty}\left(-\frac{k}{a_1}\right)^m \frac{x^{2m+1}}{(2m+1)!} = \sqrt{\frac{a_1}{k}} \sum_{m=2}^{\infty} \frac{(-1)^m \left(\sqrt{\frac{k}{a_1}}\, x\right)^{2m+1}}{(2m+1)!}$$

$$= \sqrt{\frac{a_1}{k}} \left[\sin \sqrt{\frac{k}{a_1}}\, x - \sqrt{\frac{k}{a_1}}\, x + \frac{1}{6}\left(\sqrt{\frac{k}{a_1}}\, x\right)^3\right],$$

we finally obtain

$$t_1^*(x,\tau) = T_0 - \frac{s}{\lambda k} + \frac{s}{\lambda}\tau + \left[\frac{s}{\lambda}\left(\frac{1}{ck} - \frac{c}{2a_1}\right) + \frac{1}{c}\left(b_1 - \frac{s}{\lambda}\right)\tau\right] x$$

$$+ \frac{sx^2}{2a_1\lambda} + \frac{x^3}{6a_1 c}\left(b_1 - \frac{s}{\lambda}\right) + \exp(-k\tau)\left[\frac{s}{\lambda k}\cos\sqrt{\frac{k}{a_1}}\, x - \frac{s}{\lambda}\left(\frac{1}{ck} - \frac{s}{2a_1}\right)\sqrt{\frac{a_1}{k}}\sin\sqrt{\frac{k}{a_1}}\, x\right] \tag{20}$$

We can derive precisely this solution for $t_2^*(x, \tau)$ for the standard.

Because of the selection of $A_0(\tau)$ and $A_1(\tau)$ we find that the contact conditions are satisfied. At the same time, the boundary conditions for $t_1^*(x, \tau)$ and $t_2^*(x, \tau)$ will be different from the specified boundary conditions (7) and they will therefore not be exact solutions of the formulated problem.

Applying the maximum principle to the equations of thermal conductivity, in addition to the continuous relationship between the solution and the boundary conditions, and the uniqueness of the solution, we can state that the error in the approximate solution will not exceed the error of the solution at the boundary.

Thus we have

$$|t_1(x,\tau) - t_1^*(x,\tau)| \leqslant |T_0 + b_1\tau - t_1^*(c,\tau)| \leqslant \left|\frac{c^2}{6a_1}\left(b_1 - \frac{s}{\lambda}\right)\right.$$

$$\left. + \exp(-k\tau)\left(\frac{s}{\lambda k}\cos\sqrt{\frac{k}{a_1}}\, c - \frac{s}{\lambda}\left(\frac{1}{ck} - \frac{c}{2a_1}\right)\sqrt{\frac{a_1}{k}}\sin\sqrt{\frac{k}{a_1}}\, c\right)\right|. \tag{21}$$

Given sufficiently large $\tau$, $\exp(-k\tau)$ becomes small, and for the determination of the degree of approximation in the solution we have

$$\frac{c^2}{6a_1}\left|b_1 - \frac{s}{\lambda}\right| = \frac{\lambda_2 |b_1 - b_2| c^3}{6a_1(\lambda_1 d + \lambda_2 c)} \leqslant \alpha, \tag{22}$$

where $\alpha$ is a fairly small quantity.

Condition (18) determines the greatest value of c at which the approximate solution corresponds to the specified accuracy. If $c \le d$, from (22) we can obtain the following evaluation, in approximate terms:

$$c \leqslant \sqrt{\frac{6a_1(\lambda_1 + \lambda_2)}{\lambda_2 |b_1 - b_2|}\alpha}. \tag{23}$$

We note that when $b_1 = b_2$, condition (22) is satisfied for all c, and solution (15) can therefore be set as close as you please, in this case, to the exact solution by appropriate selection of $\tau$.

Estimate (22) is valid for sufficiently large $\tau$, which are determined by the following inequality:

$$\exp(-k\tau)\left|\frac{s}{\lambda k}\cos\sqrt{\frac{k}{a_1}}\, c - \frac{s}{\lambda}\left(\frac{1}{ck} - \frac{c}{2a_1}\right)\sqrt{\frac{a_1}{k}}\sin\sqrt{\frac{k}{a_1}}\, c\right|$$

$$\leqslant \frac{s}{\lambda}\exp(-k\tau)\left[\left|\frac{1}{k}\right|\cos\sqrt{\frac{k}{a_1}}\, c\right| + \left|\frac{1}{ck}\right.$$

$$\left. - \frac{c}{2a_1}\right|\left|\sqrt{\frac{a_1}{k}}\right|\sin\sqrt{\frac{k}{a_1}}\, c\left|\right] \leqslant \frac{s}{\lambda}\exp(-k\tau)\sqrt{\frac{1}{k^2} + \frac{a_1}{k}\left(\frac{1}{ck} - \frac{c}{2a_1}\right)^2} \leqslant \beta, \tag{24}$$

where $\beta$ is a fairly small number. Hence we have

$$k\tau \geqslant \ln\frac{s}{\beta\lambda k}\sqrt{1 + a_1\left(\frac{1}{c} - \frac{c^2 k}{2a_1}\right)^2}. \tag{25}$$

For sufficiently large $\tau$

$$\max t_1^*(x, \tau) = T_0 - \frac{b}{k} - \frac{b}{2a_1}\left(\frac{c}{2} - \frac{a_1}{ck}\right)^2 + b\,\tau. \tag{26}$$

If $\beta$ is defined as some fraction of (26), for the value of $\beta$ we derive the estimate

$$\beta \leqslant \gamma\left[T_0 - \frac{b}{k} - \frac{b}{2a_1}\left(\frac{c}{2} - \frac{a_1}{ck}\right)^2\right] \leqslant \gamma T_0. \tag{27}$$

The error will differ from the solution by an order, if $\gamma \leq 0.1$.

The practical utilization of the experimental data yields a large value for $k\tau$, and condition (25) corresponds to a 3-5% error in the approximate solution.

The computational formulas for the determination of $\lambda_1$ and $a_1$ can be derived from the relationship for the temperature difference in the specimen being examined:

$$\Delta t_1 = \left(b_1 - \frac{s}{\lambda}\right)\tau - \frac{s\mu}{\lambda_1}\left[1 - \exp\left(-\frac{\lambda}{\mu}\right)\tau\right]. \tag{28}$$

Bearing in mind that the experimental data have been derived for rather high values of $k\tau$, we obtain

$$\Delta t_1 = \left(b - \frac{s}{\lambda}\right)\tau + \frac{s}{\lambda k}. \tag{29}$$

The thermograms for $t_1(c, \tau)$ and $t_1(0, \tau)$, obtained during the course of the experiment, as a rule, are straight lines in the $(t, \tau)$-plane, with various heating temperatures.

Consequently, we can also present the $\Delta t_1$ thermogram in the form of the straight line

$$\Delta t_1 = \alpha_1 + \beta_1\tau, \tag{30}$$

where $\alpha_1$ and $\beta_1$ are the straight-line parameters.

Comparing (29) and (30) and equating to each other the constant terms and coefficients for $\tau$ in the right-hand members, we derive the computational formulas for the determination of the thermal-conductivity coefficient

$$\lambda_1 = \frac{b_1 - b_2 - \beta_1}{\beta_1}\,\frac{c}{d}\,\lambda_2 \tag{31}$$

and of the thermal-diffusivity coefficient of the material being investigated, i.e.,

$$a_1 = \frac{c^2 d\,\lambda_1 (b_1 - \beta_1)\,a_2}{2a_2\,\alpha_1 (c\,\lambda_2 + d\,\lambda_1) - cd^2\lambda_2 (b_1 - \beta_1)}. \tag{32}$$

We should take note of the fact that (31) and (32) can be used only if there is a substantial difference between $b_1$ and $b_2$.

If $b_1 = b_2$, we have $\beta_1 = 0$ and formulas (31) and (32) cannot be used for the determination of $\lambda_1$ and $a_1$.

In this event we can employ another method for the derivation of the computational formulas.

Let us introduce an additional point of temperature measurement in the standard plate for the case $x = -d/2$:

$$t\left(-\frac{d}{2}, \tau\right) = T_0 + b\tau + \left[T_0 + b\tau - t(0,\tau)\right]\left[\frac{kd}{4\lambda_2}\left(\mu - \frac{d\,\lambda_2}{2a_2}\right) - \left(1 + \frac{d\,\lambda}{2c\lambda_2}\right)\right], \tag{33}$$

where $k = \lambda/\mu$. The coefficient k is found through treatment of the $T(0, \tau)$ thermogram according to the formula:

$$k = \frac{b}{\Delta t}. \tag{34}$$

The computational formula for the determination of the thermal-conductivity coefficient is derived from (33):

$$\lambda_1 = \frac{\left[\frac{d^2k}{8a_2} - \left(\frac{\Delta t_2}{\Delta t_1} + \frac{d}{2c}\right)\right]}{\frac{d}{2c}\left(\frac{d}{c} - 1\right)} \lambda_2, \tag{35}$$

where

$$\Delta t_2 = T_0 + b\tau - t_2\left(-\frac{d}{2}, \tau\right); \quad \Delta t_1 = T_0 + b\tau - t_1(0, \tau).$$

From (34) we can obtain the computational formula for the determination of the specimen's thermal-diffusivity coefficient

$$a_1 = \frac{c^2 d \lambda_1 k}{2(c\lambda_2 + d\lambda_1) - c d^2 \frac{\lambda_2}{a_2} k} \tag{36}$$

To test the accuracy of these computational formulas, we performed tests to determine the $a$ and $\lambda$ for mycalex. Plexiglas was used as the standard. The plotting and processing of the heating thermograms was accomplished in the similar method described in [3]. The basic calculation data include the following: the thickness of the mycalex plate, $c = 8$ mm; the Plexiglas thickness $d = 6$ mm; $t_1(c, \tau)$, $t_2(-d, \tau)$, $t_2(-d/2, \tau)$, and $t(0, \tau)$ are the thermograms recorded on the graph paper of the automatic recording mechanism. The specimens were heated at a constant rate of $b = 250$ deg/h.

At the instant $\tau = 14$ min, $t_1(c, \tau) = t_2(-d, \tau) = 57°C$; $t_2(-d/2, \tau) = 48°C$, $t(0, \tau) = 44°C$. The thermophysical characteristics of the standard material were $\lambda_2 = 0.16$ W/m · deg, $a_2 = 0.833 \cdot 10^{-7}$ m$^2$/sec at $t = 50°C$

From (34) we determined the coefficient $k = 20$ h$^{-1}$. From the processing of the thermograms we find $\Delta t_2 = 4°C$, $\Delta t_1 = 13°C$. Substituting the appropriate values from the original data into formula (35), we find the thermal-conductivity coefficient for the mycalex to be $\lambda_1 = 0.64$ W/m · deg at $t = 57°C$.

Simultaneously with the above-considered relative quasi-steady method we employed the absolute quasi-steady method [4] to determine the thermal conductivity of mycalex. Here we employed the theoretical formula for the determination of $\lambda$, derived from the solution of the heat-conduction equation for boundary conditions of the second kind. The heat flow set up by flat electric heaters and introduced into the specimens being investigated was measured with an automatic thermometer. The theoretical formula for the determination of $\lambda$ was simulated in a computer. The thermal conductivity was determined automatically and continuously. At $t = 57°C$ the $\lambda$ of the mycalex is 0.56 W/m · deg; as demonstrated in [4], the maximum error in the automatic determination of $\lambda$ is 8-9%.

We calculated the thermal diffusivity of the mycalex from formula (36) and at $t = 57°C$ it is equal to $a_1 = 1.86 \cdot 10^{-7}$ m$^2$/sec. To test the accuracy of this result, we performed numerous experiments to determine the $a$ of the mycalex by the method of an instantaneous heat source [5]. The thermal diffusivity of the mycalex, calculated according to this method, amounted to $2.0 \cdot 10^{-7}$ m$^2$/h.

## Analysis of the Errors of the Method

The errors in the determination of $a_1$ and $\lambda_1$ are the result of the following factors:

a) the inaccuracy of the theoretical formulas (35) and (36) resulting from the limitation to three terms in series (2) and (3), as well as from the neglect of the term $\exp(-\lambda\tau/\mu)$ in (29);

b) the inaccuracy with which the linear heating is specified;

c) the absence of reliable thermal contact between the test and standard plates;

d) the error of the sensor and of the measuring devices used in the preparation of the heating thermograms.

As follows from (23) and (27), the error indicated in item $a$ is 3%.

With an accuracy to 1%, we can neglect the term $\exp(-k\tau)$, if $k\tau \geq 4.5$. From the experimental data $k\tau = 4.67$, so that the error resulting from the neglect of the term $\exp(-k\tau)$ is equal to 0.9%. The evaluation of the errors indicated in items b, c, and d is presented in reference [4]. The total error of this method for the determination of the thermophysical characteristics amounts to 5-7%.

The method developed here is somewhat similar to the comparative quasi-steady method of Shurygina [6]; however, the physical model of the conductimeter in this method is simpler, and the method of specifying the linear heating is more universal.

## LITERATURE CITED


1. A V. Lykov, The Theory of Heat Conduction [in Russian], GITTL (1952).
2. V. V. Vlasov and N. N. Dorogov, Inzhen.-Fiz. Zh., 11, No. 3 (1966).
3. V. V. Vlasov, A. S. Klinkov, and É. O. Muratov, Trudy TIKhMa, No. 1, 120-129 (1967).
4. V. V. Vlasov, N. N. Dorogov, and V. N. Kazakov, Trudy TIKhMa, No. 1. 96-104 (1967).
5. M. V. Kulakov, Zh. Teor. Fiz., No. 3 (1952).
6. A. V. Lykov and L. Auérman, The Theory of Drying [in Russian], Moscow (1946).